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STRUCTURE AND SYMMETRY IN IANNIS XENAKIS' NOMOS ALPHA FOR CELLO SOLO

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Abstract: *We discuss the interplay between structure and symmetry in the compositional process of Iannis Xenakis' Nomos Alpha for cello (1965). Following the composer's indications provided in many of his theoretical writings, we describe the structure of the piece, which is directly provided by the combinatorial properties of a double Fibonacci process applied to a particular symmetry group. We then present our computer-aided model of the piece based on OpenMusic visual programming language.*

1 GROUP STRUCTURES IN XENAKIS' COMPOSITIONAL PROCESS

Iannis Xenakis' Nomos Alpha for violoncello is commonly considered as one of the masterpieces of the Twentieth Century repertoire for solo instruments. It represents one of the major musical realizations of what Xenakis called "symbolic music" (Xenakis 1992), a category that includes pieces based on elementary set theory (like Herma for piano), sieve-theoretical operations (like Psappha for percussion or Mists for piano), and algebraic structures (like Akrata for 16 wind instruments or Nomos Gamma for orchestra). The group-theoretical conception behind Nomos Alpha utilizes mathematical group structures as an organizing principle for musical objects, the so-called "sound complexes". These are highly structured musical objects which, like Gestalten in perception theory, are easily recognizable from a perceptual point of view. In this article we focus on the mathematical compositional process, which enables the composer to organize these sound complexes in time and within the macro structure of the piece. Figure 1 illustrates the eight prototypical "sound complexes" as described by the author and graphically represented by means of a drawing which was inspired by one of the numerous analysis of the piece (Vandeborgaerde 1968).

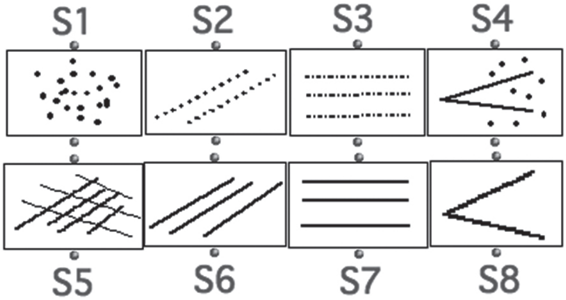

Figure 1: The eight basic sound complexes in Nomos Alpha

According with Xenakis, they represent respectively an "ataxic cloud of sound-points" (S1), a "relatively ordered ascending or descending could of sound-points" (S2), a "relatively ordered cloud of sound-points neither ascending nor descending" (S3), an "ataxic field of sliding sounds" (S4), a "relatively ordered ascending or descending field of sliding sounds" (S5), a "relatively ordered field of sliding sounds, neither ascending nor descending" (S6), an "atom represented on a cello by interferences of a quasi-unison" (S7), a "ionized atom represented on a cello by interferences, accompanied by pizzicati" (S8).

The macro-structure of the piece consists of sequences of the eight basic sound complexes and their metamorphoses, sometimes interrupted with long and almost homogeneous musical objects called "intermezzi". The order of the sound complexes is provided by means of a mathematical group, the 24 rotations that transform a cube onto itself. The eight sound complexes are attached to the eight vertices of the cube such that every single rotation determines a permutation of eight numbers, i.e. of the order of the sound complexes. Figure 2 shows the sequence of the eight abstract sound complexes attached to the group transformation D which has been chosen by the composer as the starting point for the piece, and which induces the permutation (23146758).

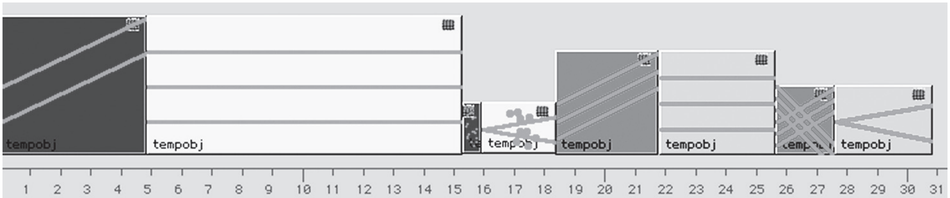

Figure 2: The first sequence of eight sound complexes in Nomos Alpha

The group structure enabled Xenakis to construct a sequence of rotations in which each element is the product of the two previous ones. Although he never defined it as such, this is clearly an example of a “Fibonacci process”. Some mathematical properties of this generalized Fibonacci process are of real interest, and may shed some light on apparently arbitrary decisions made by the composer. Firstly, concerning the intrinsic nature of that process, it turns out that it always becomes a loop. This shows the inherently cyclic nature of the process, which strictly depends on the structure of the given group. Secondly, loops may have different lengths, which refers to the total number of iterations of the Fibonacci process required to go around the loop. A Fibonacci process can never cover all 24 elements of the group: the maximal length is 18 and the largest number of different elements inside a loop is 13. We will call this number the degree of the loop. In the construction of Nomos Alpha, the Xenakis only uses Fibonacci sequences of maximal degree and maximal length.

Moreover, every series of three group transformations is associated with an additional parameter (▯, ▯ or ▯) which provides further permutations in the indices of the sound complexes associated to each vertex of the cube. The underlying compositional principle is clearly based on the search of a minimization of the redundancy (i.e. maximization of the information content). In this respect, despite Xenakis’ well-known critical position towards serial music, Nomos Alpha has many affinities with pieces based on the so-called “integral serialism”, which refers to the use of serial techniques applied to all musical parameters.

2 IMPLEMENTATION OF THE COMPOSITIONAL PROCESS

We implemented Nomos Alpha in OpenMusic, a visual programming language for computer-aided composition and music analysis, currently developed at IRCAM, Paris. One of the main characteristics of this implementation is the graphical representation of the group-theory process, together with a strong emphasis on interactivity (Agon et al. 2004). We developed a special, three-dimensional representation, helping us visualize all possible group rotations. This translates the transformations of Xenakis’ static group table into a highly dynamic object, where one can see for each element the corresponding rotation of the cube (with respect to a particular axis of symmetry) as well as the permutation induced by such a rotation. In the OpenMusic implementation, we can easily switch between the previous symbolic representation to the following audio-based representation, in which each of the sound complex of the initial segment of the score is shown in its underlying physical signal (Figure 3).

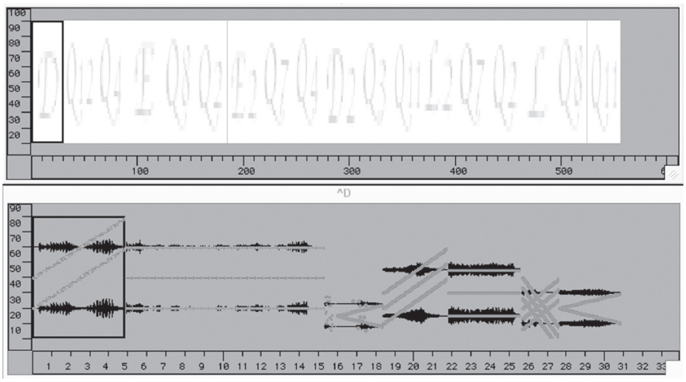

Figure 3: The symbolic/audio representation of the first eight sound complexes

3 CONCLUSIONS

In the case of Nomos Alpha, our implementation offers a general parameterized model of the compositional process with strong connections between the combinatorial properties of the underlying mathematical structures and the form of the composition. The model has both a pedagogical and a musicological interest for it enables the analyst to visualize the geometrical transformations used by the composer and to test the validity of the segmentation process and the singularity of some composer's choices. Finally, the model enables to change the initial conditions and to obtain several instances of the piece, all sharing the same stylistic properties of the published score.

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