Particle Motion Near Planets - How do They Dance Around a Magnetic Field?

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1 Introduction

Dan here - this document is coming out of discussions with Jorge in order to give us an introduction as to the three main particle motions of a particle near Earth that becomes trapped in its magnetic field. These trapped particles form a sea of particles in two doughnut shapes called the **Van Allen Radiation Belts**, which we have tried to visualise since their discovery in the late 50's/early 60's - with a few examples in figure 1. They're HUGE, they're persistent and with the growing number of satellites having to traverse them it has guaranteed that the Van Allen Radiation Belts (or just Radiation Belts) will continue to be something space scientists, satellite operators and beyond study to try and understand their current state - just like we do the weather in fact, and it's the reason their study is called **space weather**.

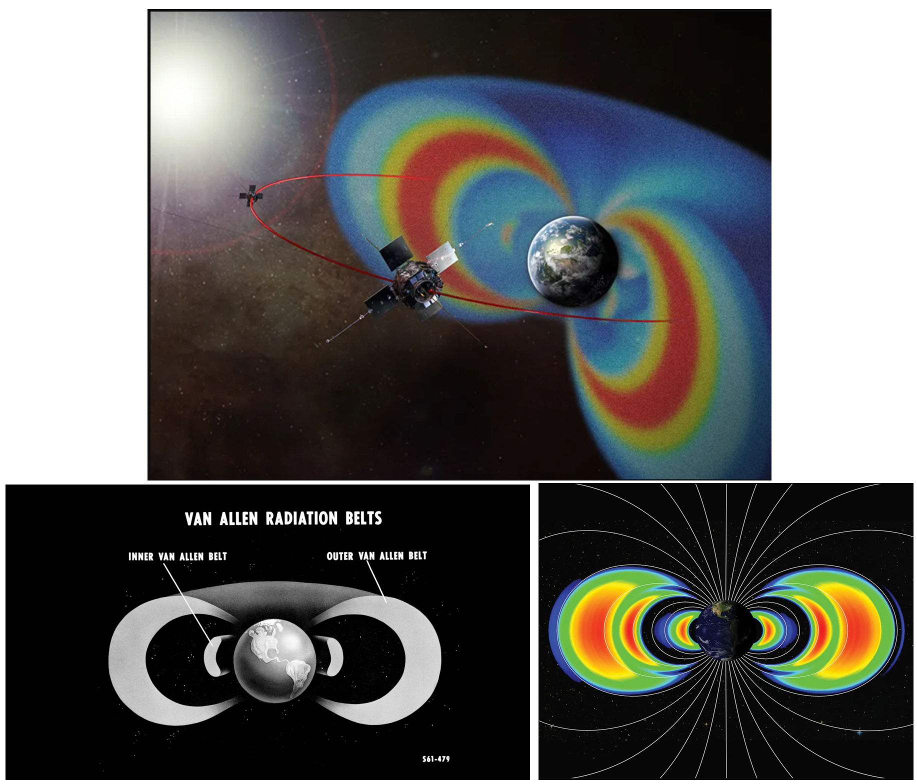

Figure 1: Visualisations of the Van Allen Radiation Belts. The colour maps in each image refer to a measure of density of particles credit: NASA Goddard

Obviously, visualising a lot of particles of different types and numbers is *a chore*, and so those who try to represent the belts visually and effectively have to make some trade-offs and decide what's important to show. One of the trade-offs is a picture of what the particles themselves are actually doing moment-by moment, which is this intricate dance of three steps - gyration, bouncing and drifting - throughout these radiation belts. A sketch of these is found in figure 2. These motions happen on different timescales, and in this document we're going to outline what these are with some science/maths details from fastest to slowest.

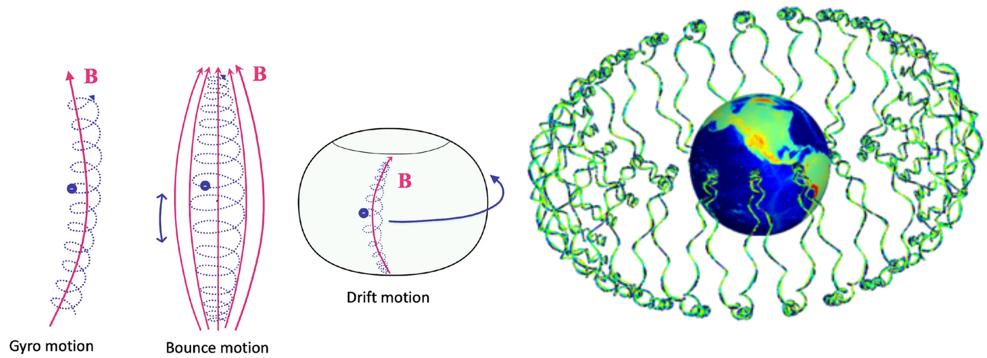

Figure 2: Left: An illustration of the three motions a particle split into the basic parts of gyration (spinning around a magnetic field line), bouncing (following a field line up and down) and drift (moving slowly between neighbouring field lines). Right: A sketch of a path of a single particle undergoing all three of these motions as it traverses Earth

2 The Fastest Dance - Gyromotion

The quickest part of a particle's dance is the circle motion it makes around a magnatic field line, an invisible line representing the magnetic field of the planet. Because charge is present in the system, the motion of charged particles

On this smallest, fastest scale, charged particles wind in a helical fashion around one of these field lines. The speed at which these do so is given by the **gyrofrequency** for the particle:

$$\Omega = \frac{qB}{m}\,, \tag{1}$$

where q is the **charge** of the particle, B is the **magentic field strength** of the magnetic field line it's spinning around and m is the **mass** of the particle. The gyrofrequency for a particle in the Van Allen radiation belts (so several Earth radii's distance out! Let's take this to be 3 Earth radii here) the parameters in the gyrofrequency for an electron are

$$q = -1.6 \times 10^{-19}\,, \quad B = 3 \times \frac{1}{3^3} \times 10^{-5} \approx 10^{-6}\,, \quad m = 9.1 \times 10^{-31}$$

and so the gyro frequency is

$$\Omega \approx -10^5 \text{ Hz}$$

and corresponding time period

$$T = \frac{2\pi}{|\Omega|} \approx 6 \times 10^{-5} \text{ sec}$$

This is obviously MUCH TOO HIGH a frequency to resolve with audio equipment I think, but it gives us a baseline because this is the fastest/highest frequency. We can however use this as our 'unit of time' to have the rest of the motion evolve - such as 'the second and third motions happen over 100 or 1000 gyroperiods.

The final comment is how we modify this for energetic, **relativistic** particles. This starts of rather simple with a new factor dividing the gyrofrequency:

$$\Omega = \frac{qB}{m\gamma}$$

where γ is the **Lorentz factor**, which can be defined as the ratio between a particle's actual energy E compared to a particle's rest energy E_0:

$$\gamma = \frac{E}{E_0}\,, \quad E_0 = 8.187 \times 10^{-14}\,.$$

We have that $E \geq E_0$. This allows us to decrease the gyrofrequency should we need to by up to a factor of 4 (physically, but can stretch reality if we need to) by setting γ up to a value of 4.

3 The Intermediate Shimmy - Bounce Motion

As the particle circles a field line, there is a much slower motion that takes place - the slow climb or descent along the field line. A cartoon of how this might look is presented below in figure 3. What happens over this motion is that a particle will travel up or down the field line until it reaches a **mirror point**, which does exactly what the name implies - it reflects a particle back up/down the field line. Ultimately, the particle will ping-pong between these mirror points until something frees the particle from it's magnetic prison[1]

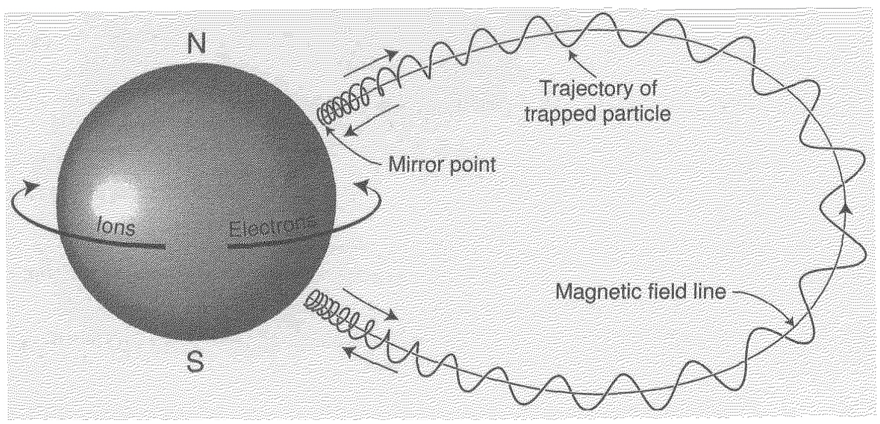

Figure 3: An illustration of gyromotion and bounce motion along one of Earth's field line - the spiral/helical motion is the gyromotion, that gets more compressed near a mirror point, where a particle bounces back up/down a field line. Source: Baumjohan and Treumann, *Basic Space Plasma Physics*, 1996 (far from a basic book, the irony...)

How often does it bounce? The answer is hard, the mathematical version of 'it depends' - a good estimate for this (for our purposes) is the expression (see Baumjohann and Treumann (2012), pg. 36)

$$\Omega_{bounce} = \frac{2\pi}{T_{bounce}} \approx \frac{12}{r}\sqrt{\frac{E}{m}}\,,$$

where r is the distance of the particle from the Earth when it's at the magnetic equator (so in line with the Earth's equator, measured outwards) and E is the **energy** of the particle in question. This tells us higher energy particles and those close to the Earth bounce faster (as expected, maybe?) and those further from the Earth or with lower energy bounce more slowly. To get a handle on the size of this we again use some typical parameters for three Earth radii away - these can be varied depending on energy, but let's take

$$r = 3R_E \approx 2 \times 10^7\,, \qquad m = 9.1 \times 10^{-31}\,, \qquad E = 1 \times 10^{-13}\text{Joules}$$

to give

$$\Omega_{bounce} \approx 6 \times 10^{-7}\sqrt{\frac{1}{9} \times 10^{18}} \approx 2 \times 10^2 \text{ Hz}$$

[1]Things that can free our subatomic prisoner can include: waves 'boosting' the particle fast enough that it crashes through a mirror point or a geomagnetic storm shunting a field line away, leaving poor particle looking for a new home

and so the period is around $\pi \approx 31$ milliseconds. This is 100 times slower than gyromotion!

How far does this go up a field line? I think this is less important for a demo - **I would be inclined to leave this aspect open to artistic liberty**.

4 The Last Waltz - Drift motion

As the particle is gyrating around and bouncing along field lines, the third procession is slowly taking place - drift. This process happens over many bounce periods and corresponds to a particle moving across to different magnetic field lines and bouncing up and down them as it goes. In this way, it slowly orbits the planet, a process which can take minutes or hours to complete. Turning to Baumjohann and Treumann (2012) we have an approximation to the scale of this thing:

$$\Omega_{drift} = \frac{3Er}{2\pi q B R_E^3}$$

As before, let's see what value this takes for the previous parameters:

$$r = 3R_E \approx 2 \times 10^7\,, \qquad E = 1 \times 10^{-13}\text{Joules}\,, \qquad q = -1.6 \times 10^{-19}\,, \quad B = 3 \times \frac{1}{3^3} \times 10^{-5} \approx 10^{-6}\,,$$

gives

$$\Omega_{drift} \approx 0.16Hz$$

and so the period of motion is $2\pi/0.16 \approx 40$ seconds. This is around 1000 times slower than bounce motion in this case!

A useful thing to track will be the ratio between this frequency and the next fastest one, the bounce frequency. This ratio is

$$\frac{\Omega_{drift}}{\Omega_{bounce}} = \frac{\sqrt{mE}r^2}{8\pi q B R_E^3} \approx 1.3 \left(\frac{r}{R_E}\right)^5 \sqrt{E}\,. \tag{2}$$

The large bracket may seem intimidating, but it basically becomes a small number given we measure the radius from Earth in Earth radii. What makes the ratio small is the energy, which as we've seen is very small, and so tends to lead to a large gap between the two frequencies. Why compute this? Well, it allows us to **anchor the drift frequency to a choice of bounce frequency in later usage**

5 Summary and suggestions

To wrap up, our motion is composed of three oscillatory motions which operate at three much seperate rates. Gyromotion is exceedingly rapid, bounce motion moderate and drift motion by far the slowest.

Dan suggests the following: One can almost directly translate these motions into a sound piece as follows:

- Consider a spherical co-ordinate system, characterised by **radial co-ordinate** R, latitude co-ordinate Θ (the curved up-and-down with zero representing the equator) and Φ the azimuthal (i.e. around-the-equator drift co-ordinate rather than the up-and-down bounce one). A sketch of polar co-ordinates roughly equationg to this is in figure 4
- Choose how far we would like the particle/sound to be from the speaker - r, say - this will be our R value that will not change in time [2]
- With this in mind, the latitude co-ordinate of our particle will be an oscillatory up-and-down with frequency Ω_{bounce} with maximum deflection we can choose - let this maximum latitude/amplitude be denoted by A. Then our latitude co-ordinate would be

$$\Theta = A\cos(\Omega_{bounce} t)\,,$$

where t is time in seconds

[2] There's a more accurate way to determine this, but walk before we run - if we want something more 'accurate' later Dan will revisit this

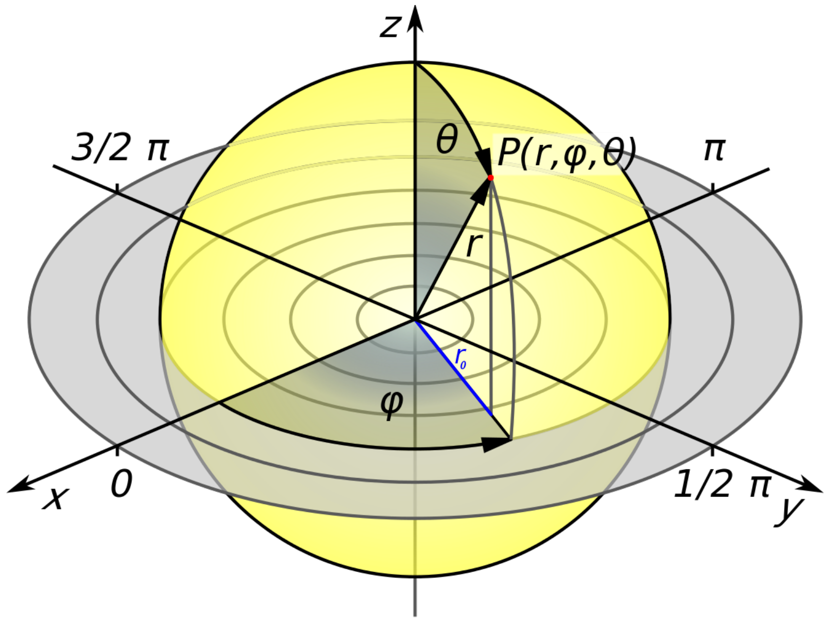

Figure 4: A sketch of spherical polar co-ordinates. NOTE THAT IN THIS SKETCH, θ is NOT MEASURED FROM THE EQUATOR AS PER DISCUSSION IN THIS SECTION!!

- Finally, the speed at which we rotate around the 'Earth' (read: IKO) is a distance $\times$ frequency, which is our radial distance times the drift frequency, so $r\Omega_{drift}$. As this is constant in time, this means our azimuthal position Φ is speed multiplied by time:

$$\Phi = r\Omega_{drift}t$$

- Thus, at any instance of time, the location of our particle/sound point could be given by

$$(R, \Theta, \Phi) = (r, A\cos(\Omega_{bounce}t), r\Omega_{drift}t)$$

- Now, instead of using the direct frequencies (which would be absolutely buck wild in terms of an experience!), we can instead choose a Ω_{bounce} that works/is experiencially satisfying and use equation (2) to determine what the drift frequency that corresponds to that bounce frequency, choosing suitable parameters for the energy to control the drift frequency 9so it doesn't take 5 minutes for a particle/sound to return, say) as necessary.
- As for the sound profile of the localised sound, maybe we could encode either the gyrofrequency in some way (higher frequency = higher pitch, normalised to normal hearing range) or something similar using γ.

References

W. Baumjohann and R. A. Treumann. *Basic space plasma physics.* World Scientific, 2012.